

The Theory of the Moiré Phenomenon

by

Isaac Amidror

*Peripheral Systems Laboratory,
Ecole Polytechnique Fédérale de Lausanne,
Lausanne, Switzerland*



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If the impulse whose frequency vector is \mathbf{f} falls inside the visibility circle and represents a visible moiré in the superposition of the m original images, the above formulas (2.8) express the frequency, the period and the angle of this moiré. Note that, as shown in Sec. C.1 of Appendix C, in the special case of $m = 2$, where a moiré effect occurs due to the vectorial sum of the frequency vectors \mathbf{f}_1 and $-\mathbf{f}_2$ (see Fig. 2.2), these formulas are reduced to the familiar geometrically obtained formulas of the period and angle of the moiré effect between two gratings [Nishijima64]:¹¹

$$T_M = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2 - 2T_1 T_2 \cos \alpha}} \quad \varphi_M = \arctan \left(\frac{T_2 \sin \theta_1 - T_1 \sin \theta_2}{T_2 \cos \theta_1 - T_1 \cos \theta_2} \right) \quad (2.9)$$

where T_1 and T_2 are the periods of the two original gratings and α is the angle difference between them, $\theta_2 - \theta_1$. (Note, however, that these formulas are only valid when $T_1 \approx T_2$; the reason for this restriction will become clear at the end of Sec. 2.6.) In the particular case where $T_1 = T_2$ this is further simplified into the well-known formulas [Nishijima64]:

$$T_M = \frac{T}{2 \sin(\alpha/2)} \quad \varphi_M = 90^\circ + \frac{1}{2}(\theta_1 + \theta_2) \quad (2.10)$$

Note that in this case the direction of the moiré is perpendicular to the bisector of the angle formed between the grating directions. In another interesting particular case where $T_1 \neq T_2$ but $\theta_1 = \theta_2$ (namely: the superposition of two parallel gratings) Eqs. (2.9) are reduced into the equally well-known formulas:

$$T_M = \frac{T_1 T_2}{|T_2 - T_1|} \quad (\text{i.e., } f_M = |f_2 - f_1|), \quad \varphi_M = \theta \quad (2.11)$$

Eqs. (2.7), (2.8) and their derived formulas (2.9)–(2.11) give the *geometric properties* of an impulse in the spectrum of the superposition (and of the periodic component or moiré that it represents in the image domain), namely, the period and the direction. The *amplitude* of any individual impulse, which represents the strength of the corresponding periodic component in the image, is a product of the amplitudes of the m impulses from which it has been obtained in the convolution, one from each of the m spectra:

$$a = a_1 \cdot \dots \cdot a_m \quad (2.12)$$

Note, however, that if two or more impulses in the convolution happen to fall on top of each other exactly in the same location, their individual amplitudes are summed.

As we can see from Eqs. (2.7) and (2.12), the convolution of impulsive spectra can be considered as an operation in which frequency vectors of the original spectra are added vectorially, whereas the corresponding impulse amplitudes are multiplied. These rules follow from the properties of convolution, and they can be readily verified by the “move and multiply” method. Note that if all the convolved spectra are real-valued and symmetric about the origin (see Sec. 2.2), the resulting spectrum is also real-valued and symmetric,

¹¹ Note that the moiré angle formulas found in literature may vary according to the angle conventions being used.

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- [Nishijima64] Y. Nishijima and G. Oster, "Moiré patterns: their application to refractive index and refractive index gradient measurements," *Jour. of the Optical Society of America*, Vol. 54, No. 1, January 1964, pp. 1-5.
- [Ohyama86] N. Ohyama, M. Yamaguchi, J. Tsujiuchi, T. Honda and S. Hiratsuka, "Suppression of moiré fringes due to sampling of halftone screened images," *Optics Communications*, Vol. 60, No. 6, December 1986, pp. 364-368.
- [Oster63] G. Oster and Y. Nishijima, "Moiré patterns," *Scientific American*, Vol. 208, May 1963, pp. 54-63.
- [Oster64] G. Oster, M. Wasserman and C. Zwerling, "Theoretical interpretation of moiré patterns," *Jour. of the Optical Society of America*, Vol. 54, No. 2, 1964, pp. 169-175.
- [Oster65] G. Oster, "Optical art," *Applied Optics*, Vol. 4, No. 11, 1965, pp. 1359-1369.
- [Oster69] G. Oster, *The Science of Moiré Patterns*. Edmund Scientific Co., Barrington, NJ, 1969 (second edition).
- [Ostromoukhov93] V. Ostromoukhov, "Pseudo-random halftone screening for color and black&white printing," Proceedings of the 9-th International Congress on Advances in Non-impact Printing Technologies, Yokohama, October 1993, pp. 579-582; also reprinted in *Recent Progress in Digital Halftoning*, R. Eschbach (Ed.), IS&T, 1994, pp. 130-134.
- [Ostromoukhov95] V. Ostromoukhov and R. D. Hersch, "Artistic screening," Proceedings SIGGRAPH 95, Computer Graphics Proceedings, Annual Conference Series, 1995, pp. 219-228.
- [Papoulis68] A. Papoulis, *Systems and Transforms with Applications in Optics*. McGraw-Hill, NY, 1968.
- [Patorski76] K. Patorski, S. Yokozeki and T. Suzuki, "Moiré profile prediction by using Fourier series formalism," *Japanese Jour. of Applied Physics*, Vol. 15, No. 3, 1976, pp. 443-456.
- [Patorski93] K. Patorski, *Handbook of the Moiré Fringe Technique*. Elsevier, Amsterdam, 1993.
- [Post67] D. Post, "Sharpening and multiplication of moiré fringes," *Experimental Mechanics*, Vol. 7, April 1967, pp. 154-159.
- [Post94] D. Post, B. Han and P. Ifju, *High Sensitivity Moiré: Experimental Analysis for Mechanics and Materials*. Springer-Verlag, NY, 1994.
- [Pratt91] W. K. Pratt, *Digital Image Processing*. John Wiley & Sons, NY, 1991 (second edition).
- [Pssc65] Physical Science Study Committee, *Physics*. D.C. Heath and Company, Boston, 1965 (second edition).
- [Renesse98] R. L. van Renesse (Ed.), *Optical Document Security*. Artech House, Boston, 1998 (second edition).
- [Réveillès91] J.-P. Réveillès, *Géométrie Discrète, Calcul en Nombres Entiers et Algorithmique*. Thèse d'Etat, Université Louis-Pasteur, Strasbourg, 1991 (in French).
- [Rodriguez94] M. Rodriguez, "Promises and pitfalls of stochastic screening in the graphics arts industry," Proceedings of the 47th Annual Conference of the IS&T, May 1994; also reprinted in *Recent Progress in Digital Halftoning*, R. Eschbach (Ed.), IS&T, 1994, pp. 34-37.
- [Rogers77] G. L. Rogers, "A geometrical approach to moiré pattern calculations," *Optica Acta*, Vol. 24, No. 1, 1977, pp. 1-13.