of the carrier and tap multipliers both have Gaussian shapes. Then the autocorrelation function can be written.

Bandpass Carrier Autocorrelation:

$$R_s(\tau) = e^{-(\pi\tau W)^2}\cos\omega_0\tau \qquad \text{for } G_s(f) \sim \exp-(f-f_0)^2/W^2 + \exp-(f+f_0)^2/W^2.$$

Tap Multiplier Autocorrelation:

$$R(\tau) = e^{-(\pi\tau B)^2} \qquad \text{for } G(f) \sim e^{-(f/B)^2}.$$

Thus, using (6), we have a time offset degradation factor

$$F_t(T_1,T_0) = \exp\left\{-\pi^2W^2\left[\frac{B^2}{W^2}T_1^2 + (T_1 - T_0)^2\right]\right\} \cos\omega_0(T_0 - T_1).$$

It is easily seen that the envelope of this function has a maximum with receiver delay[3]

$$T_1 = T_0W^2/(B^2 + W^2) < T_0$$

and the maximum of the envelope is

$$F_{tm}(T_0) = \exp -\pi^2\left[\frac{B^2W^2}{B^2 + W^2}T_0^2\right].$$

Hence, for these spectra, this figure of merit decreases exponentially with the delay offset.

Frequency Offset Effect

In order to evaluate the effect of the frequency offset and channel time dispersion, we consider two example sets of tap multiplier coefficients.

1) Uniform tap variances: $a_k = a$ if $|k| \leq N$ and $a_k = 0$ if $|k| > N$.

2) Decaying tap variances: $a_k = a/(k^2 + M^2)$.

The summation $F_f(\omega_\Delta)$ contains the basic information giving the attenuation of the useful signal energy caused by the channel time dispersion.

For uniform tap variances, we have

$$F_f(\omega_\Delta) = \frac{1}{2N+1}\sum_{-N}^{N}\cos k\omega_\Delta T = \left(\frac{1}{2N+1}\right)\left[\frac{2\cos\{[(N+1)/2]\omega_\Delta T\}\sin(N\omega_\Delta T/2)}{\sin(\omega_\Delta T/2)} + 1\right] = \frac{1}{2N+1}\,\frac{\sin\theta[(2N+1)/2N]}{\sin(\theta/2N)}$$

where we define $T_D \triangleq NT$ as the time dispersion of the channel, and $\theta \triangleq \omega_\Delta T_D$ as the phase difference for this frequency offset and dispersion time. This variation in performance with θ is plotted in Fig. 1 for several values of N. Note the attenuation in F_f as θ increases above unity.

Consider now a channel represented by an infinite number of taps having decaying variance. The summation can now be written [12]

$$F_f(\omega_\Delta) = \sum\frac{\cos k\omega_\Delta T}{k^2 + M^2}\Big/\sum\frac{1}{k^2 + M^2} = \frac{\cosh M(\pi - \omega_\Delta T)}{\cosh \pi M} \text{ for } 0 < \omega_\Delta T < 2\pi.$$

This expression is plotted in Fig. 2 for several values of M. For a large channel dispersion-carrier bandwidth product $M \gg 1$, we have the approximate result

$$F_f(\omega_\Delta) \cong \exp -\theta \quad \text{where } \theta \triangleq \omega_\Delta T_D,\ T_D \triangleq MT.$$

Again, substantial degradation occurs as θ increases above unity.

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[3] If we assume that $\omega_0 >> W, B$, then this T_1 is approximately optimum, and the maximum of the envelope is the approximate maximum of the function. For a true maximum we, of course, want $\omega_0(T_1 - T_0) \cong 2n\pi$.

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Correction to "Experiments in 'SSB FM' Communication Systems"

There is an error in the above paper.[1] Figure 15 was printed over its own caption and also over the caption for Fig. 16. Figure 16 was not printed at all. The correct figures and their captions are as shown.

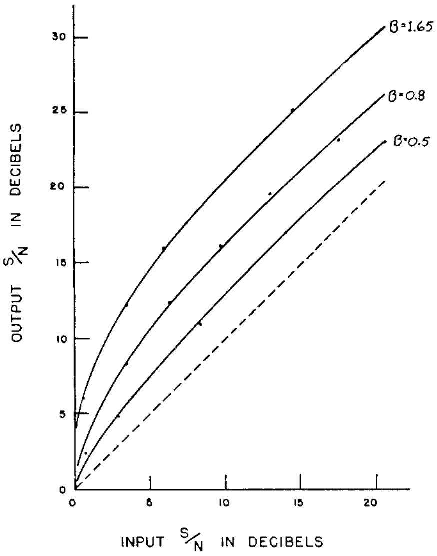

Fig. 15.

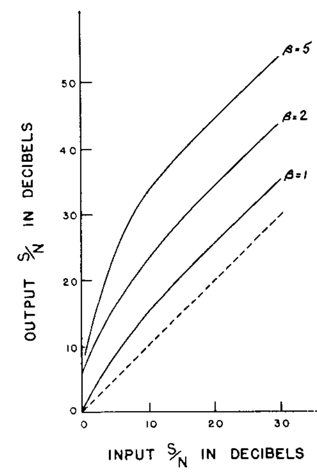

Fig. 16.

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Manuscript received July 2, 1965.
[1] *IEEE Trans. on Communication Technology* vol. COM-13, pp. 109–116, March 1965.

On Survivability of Networks

Narsingh Deo suggests an alternative metric of survivability[1] (pairs of stations in communications) to the one I chose to use in a recent paper[2] on survivable communications (number of stations in communications with all). His point is well

Manuscript received May 21, 1965.
[1] N. Deo, "On survivability of communications systems," *IEEE Trans. on Communication Technology (Correspondence)*, vol. COM-12, pp. 227–228, December 1964.
[2] P. Baran, "On distributed communications networks," *IEEE Trans. on Communication Systems*, vol. CS-12, pp. 1–9, March 1964.

stated and I would like to add to his remarks.

The most appropriate metric of survivability depends chiefly on what you want the network to do. I discussed a large strategic communications network undergoing attack. Here, the penalty for noncentralized, divergent action differs from the case of a tactical network where clumps of forces may carry on even though cut off from a point of central authority.

Thus, I use the pessimistic measure of considering only forces connected to the largest single group of surviving stations—with a tacit assumption that the command element is mobile or distributed. Those strategic elements not connected to the largest surviving group might (from the viewpoint of ease of terminating the outbreak of hostility) best await restoration of communications before taking irrevocable or complicating actions.

I concur that the metric Deo uses is more appropriate to the tactical theater where the penalty of being cut off from central authority is of less concern.

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Automatic Equalization Using Transversal Equalizers

Some interesting articles on automatic, transversal equalization [1], [2] have appeared in recent months. This subject is considered highly important.[1] One purpose of this correspondence is to point out some earlier, less widely circulated reports that discuss this subject in considerable detail and explain some additional, important considerations.

The type of equalizer under discussion consists basically of a delay line with adjustable taps. The general method of automating this equalizer described in recent references [1], [2] as well as by the writer [3]–[5] is based upon time-domain equations derived from the system pulse response by using simple superposition [3], [4]. This general method is interesting in theory, but has the following important disadvantages:

1) It depends upon utilizing samples of the system pulse response, including rather small parts of this response, in the presence of noise.

2) In order to measure or utilize these sample amplitudes, it is necessary to interrupt the flow of regular data long enough to transmit isolated pulses.

For automating the equalizer, a scheme that offers some important practical advantages has been described [5]. Briefly, a conventional "eye pattern" [2]–[4] (formed from the regular baseband signal on the face of an oscilloscope) is electronically scanned. Each potentiometer of the equalizer is automatically adjusted for the best "eye" opening, the signal from the scanner being used as an error signal in a servo-type arrangement used to drive the potentiometer. The potentiometers are adjusted sequentially in the order of decreasing expected effect upon the eye pattern. The advantages of this approach include the following:

1) The equalizer can be automatically adjusted in the presence of relatively poor signal-to-noise and high pre-equalization distortion.

2) The measurement errors can be largely averaged out, thus increasing the accuracy of equalization.

3) Equalization is from the regular baseband signal alone. There is no need to transmit special pulses for equalization. The equalization process can proceed continually while data is being transmitted. The latter advantage could possibly make it feasible to correct time-varying, frequency-selective fading, such as that caused by multipath in some types of radio systems. The oscilloscope can be replaced by basically similar methods of scanning the baseband signal.

The same reference [5] also describes some variations of this method of implementing the automation of the equalizers, including a method for use when the transmitted signal is unknown and there is no opening of the "eye pattern" prior to equalization.

This type of equalizer with manual adjustments has been used since early 1961 in the first multiple-level, vestigial sideband, suppressed carrier, data modem[2] and has proved highly versatile and accurate [4], [5].

Some examples of other important considerations previously discussed in detail [3]–[5] are:

1) Time-domain analyses which show a number of important relationships, including matrices inherently well suited for digital implementation of the automatic equalization.

2) Frequency-domain analyses which show, for example, that when the phase-frequency curve nonlinearity can be approximated by a sine wave, $B \sin g\,\omega$, the proper settings of the potentiometers are:

$$G_0 = 2J_0(B)$$

$$G_n = G_{-n} = J_n(B), \quad n \text{ even}$$

$$G_n = G_{-n} = -J_n(B), \quad n \text{ odd},$$

where $J_n(B)$ is the Bessel function of the first kind of order n and G_n is the gain setting for the nth tap numbered from the middle of the delay line. These analyses also show the high versatility obtainable from this type of equalizer, as well as estimates of the range and accuracy obtainable from limited numbers of stages [3], [4].

3) The use of the same type of equipment for combating impulse noise [4].

4) Determination of critical combinations of bits or other signaling elements, i.e., those resulting in the maximum distortion, or maximum positive and negative error in signal amplitude [3].

5) Schemes for ganging the potentiometers to reduce the number of adjustments under certain conditions [3].

6) Use of a shift register to replace the delay line, especially for shaping the signal prior to transmission [3].

7) Considerations involved in letting the sampling interval equal the signal element during ("inter-bit adjustment" equalization) [3].

8) The number of equalizer stages required for some typical applications. For example, in agreement with results now reported by Rappeport [1], six stages were reported as adequate for most telephone line applications [3].

The writer can offer interested readers copies of [3]–[5] to a limited extent

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Manuscript received April 2, 1965.

[1] Automatic equalization has sometimes appeared on the military's list of most wanted inventions, and the volume of data transmission over facilities requiring accurate, versatile equalization is rapidly increasing.

[2] Four ACF Industries patents by E. D. Gibson pending on the modem and equalizer. This was the first known data modem capable of achieving 4800 bits per second over a wide variety of ordinary telephone lines, and this is the same type of application discussed in the recent references [1], [2].

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